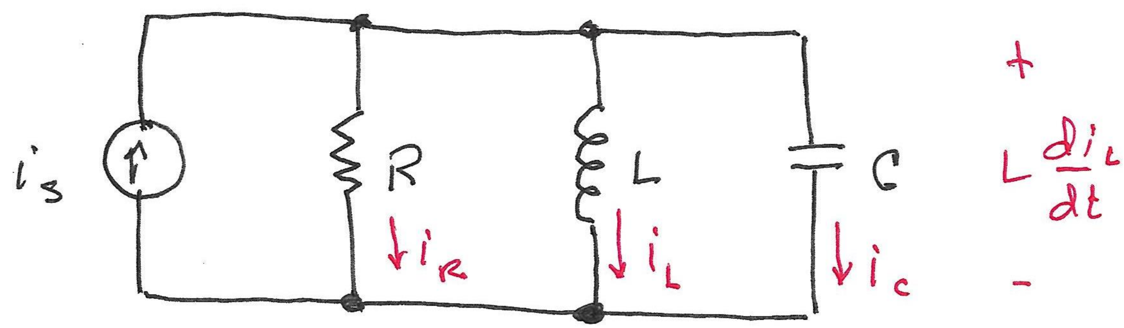
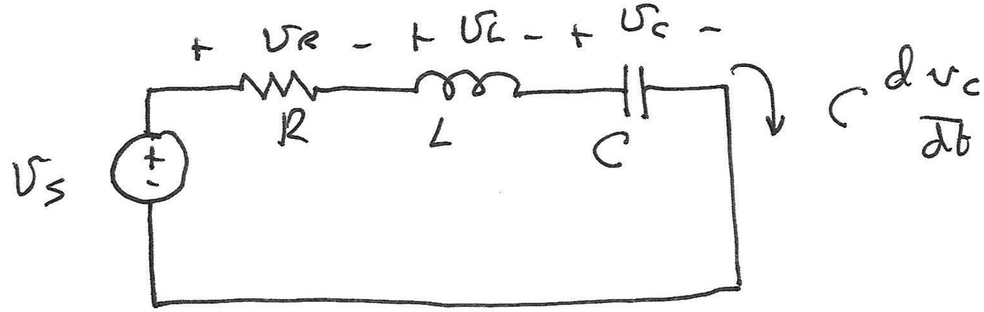


$$i_R = \frac{1}{R} L \frac{di_L}{dt}$$

$$i_c = C \frac{d}{dt}\left[L \frac{di_L}{dt}\right] = CL \frac{d^2 i_L}{dt^2}$$

$$i_c + i_R + i_L = i_s$$

$$CL \frac{d^2 i_L}{dt^2} + \frac{L}{R}\frac{di_L}{dt} + i_L = i_s$$



$$v_R = RC\frac{dv_c}{dt}$$

$$v_L = L\frac{d}{dt}\left(C\frac{dv_c}{dt}\right) = LC\frac{d^2 v_c}{dt^2}$$

$$LC\frac{d^2 v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = v_s$$

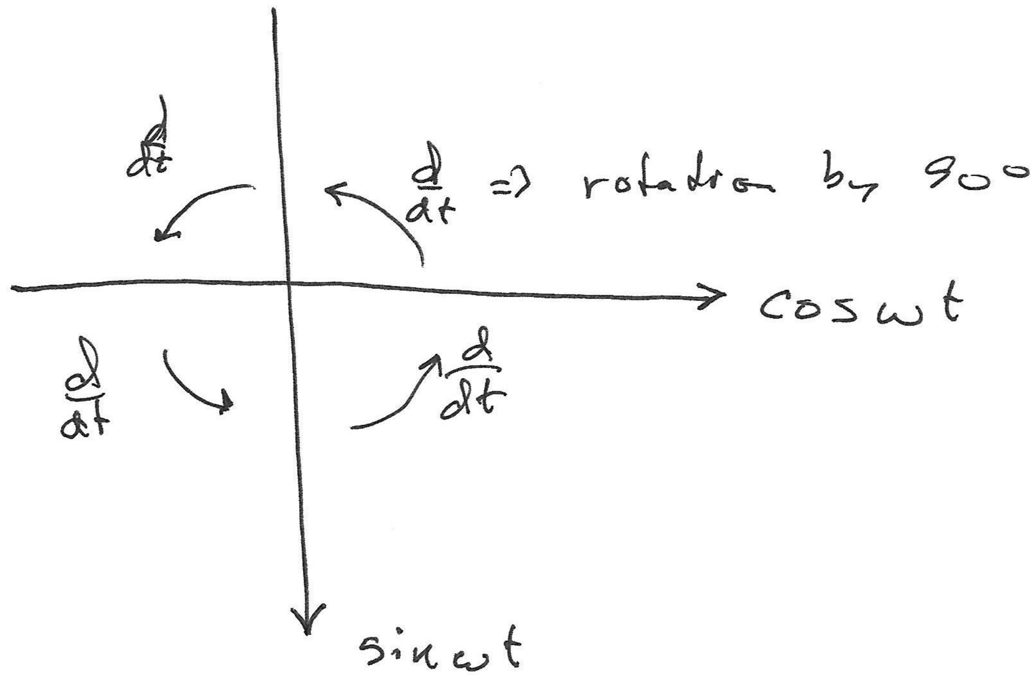
AC circuit analysis

$\Downarrow$ sources are sinusoidal

$$v_s = 177\cos(2\pi\, 60 t + \theta)$$

$$LC \frac{d^2 v_c}{dt^2} + RC \frac{dv_c}{dt} + v_c = v_s$$

C sin ωt   B cos ωt   A sin ωt $\Uparrow$ sin ωt


$\frac{d}{dt}$ ⇒ rotation by 90°
$\frac{d}{dt}$
$\frac{d}{dt}$
$\frac{d}{dt}$
$\cos\omega t$
$\sin\omega t$

Complex Numbers

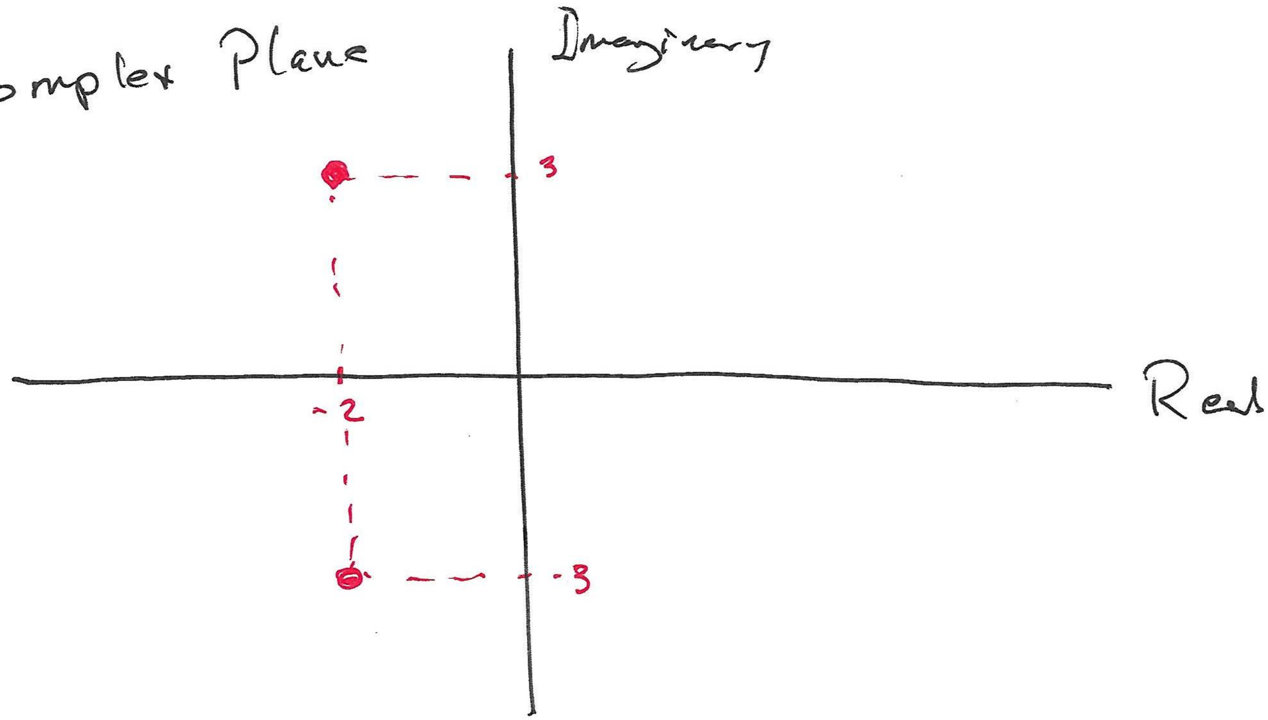
$$x^2 + 4x + 13 = 0$$

$$(x+2)^2 + 3^2 = 0$$

$$x = -2 \pm j3$$

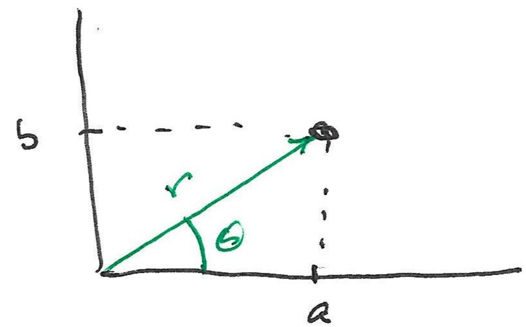
$$\boxed{\begin{aligned} x &= \frac{-4 \pm \sqrt{4^2 - 4\cdot 1\cdot 13}}{2\cdot 1} \\ &= \frac{-4 \pm \sqrt{16-52}}{2} \\ &= \frac{-4 \pm \sqrt{-36}}{2} \quad = \sqrt{-1}\,\sqrt{36} = 6j \\ &= \frac{-4 \pm 6j}{2} \\ &= -2 \pm 3j \end{aligned}}$$

Complex Plane



Let $z = a + jb$ — Cartesian Form or Rectangular Form

Real Part — Imaginary Part



$$a = r\cos\theta$$

$$b = r\sin\theta$$

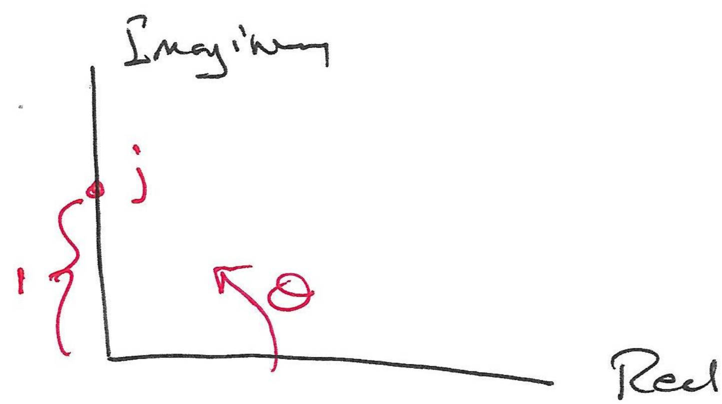
$$z = r\cos\theta + jr\sin\theta$$

Euler's Identity

$$r\,e^{\pm j\theta} = r\cos\theta \pm rj\sin\theta$$ Exponential Form

$$r\angle\theta$$ Polar Form
Phasor

$$r\angle\theta \equiv re^{j\theta} \equiv r(\cos\theta + j\sin\theta)$$



$$j = 1\angle 90^\circ$$

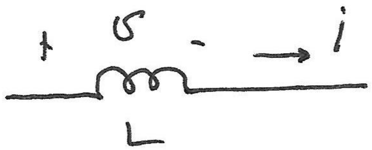
Multiply a complex number by j.

$$A\angle\theta \cdot j = A\angle\theta \cdot 1\angle 90^\circ$$

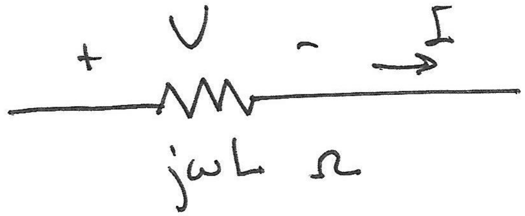
$$= A e^{j\theta} \cdot 1 e^{j90^\circ}$$

$$= A e^{j(\theta + 90^\circ)}$$

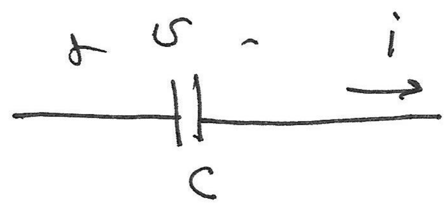
$$= \underline{A \angle \theta + 90^\circ}$$



$$v = L\frac{di}{dt} \quad \Rightarrow \quad V = L\,(j\omega I)$$

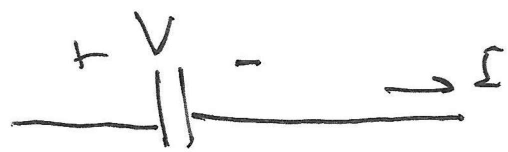


Impedance

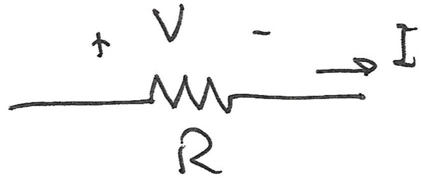


$$i = C\frac{dv}{dt} \Rightarrow I = C\,j\omega V$$

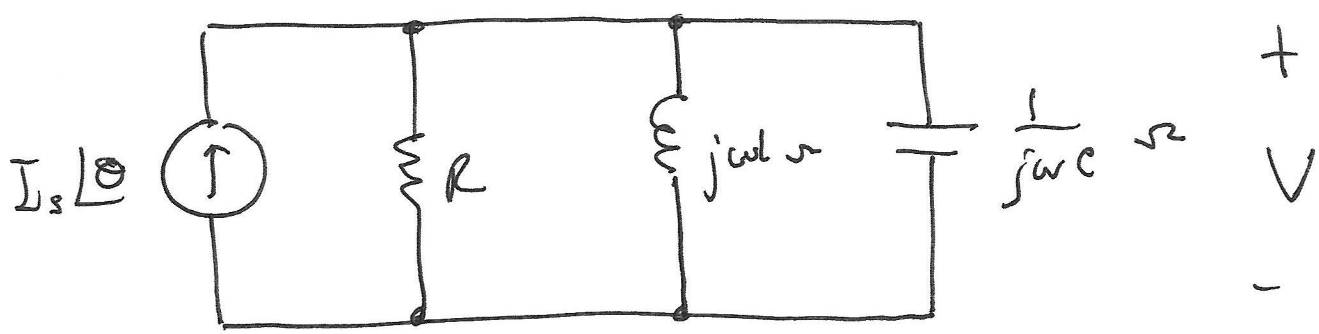
$$V = \frac{1}{j\omega C} I$$



$\frac{1}{j\omega C}\ \Omega$



$$V = RI$$



$$\frac{V}{R} + \frac{V}{j\omega L} + \frac{V}{\left(\frac{1}{j\omega C}\right)} = I_s \angle\theta$$

$$\left(\frac{V}{R} + \frac{V}{j\omega L} + j\omega C\, V\right) = I_s \angle\theta$$

$$-\omega^2 LC$$

$$\frac{j\omega L + R + (j\omega L)(j\omega C)}{j\omega L \cdot R} V = I_s \angle\theta$$

$$\frac{(R - \omega^2 LC) + j\omega L}{j\omega LR} V = I_s \angle\theta$$

$$V = \frac{j\omega LR}{(R - \omega^2 LC) + j\omega L} I_s \angle\theta$$

Let $z_1 = a + jb$

$z_2 = c + jd$

$$z_1 + z_2 = a + jb + c + jd = (a + c) + j(b + d)$$

$z_1 = 1 + j2$

$z_2 = 3 - j5$

$$z_1 + z_2 = 1 + j2 + 3 - j5 = 4 - j3$$

$$z_1 = a + jb = A_1 \angle \theta_1$$

$$z_2 = c + jd = A_2 \angle \theta_2$$

$$z_1 \cdot z_2 = (a + jb)(c + jd)$$

$$= ac + jad + jbc - bd$$

$$= (ac - bd) + j(ad + bc)$$

$$z_1 \cdot z_2 = A_1 \angle \theta_1 \cdot A_2 \angle \theta_2$$

$$= A_1 A_2 \angle \theta_1 + \theta_2$$

$$z_1 = a + jb = A_1 \angle \theta_1$$

$$z_2 = c + jd = A_2 \angle \theta_2$$

$$\frac{z_1}{z_2} = \frac{a + jb}{c + jd} \cdot \frac{c - jd}{c - jd}$$

$$= \frac{(a + jb)(c - jd)}{c^2 \cancel{- jcd} \cancel{+ jdc} + d^2}$$

$$= \frac{(a + jb)(c - jd)}{(c^2 + d^2)}$$

$$= \frac{ac - jad + jbc + bd}{c^2 + d^2}$$

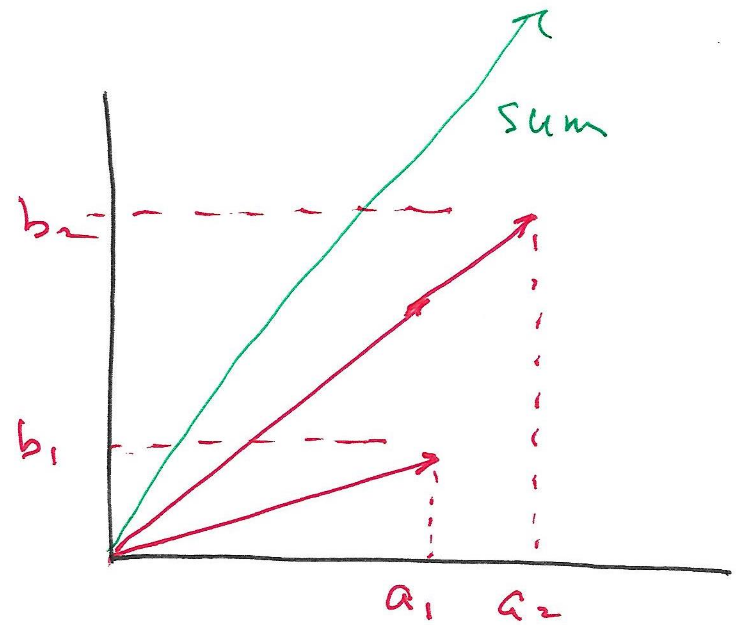
$$= \frac{(ac + bd) + j(bc - ad)}{c^2 + d^2}$$

$$\frac{z_1}{z_2} = \frac{A_1 \angle \theta_1}{A_2 \angle \theta_2} = \frac{A_1}{A_2} \angle \theta_1 - \theta_2$$

$$A_1 e^{j\theta_1} \cdot A_2 e^{j\theta_2} = A_1 A_2 e^{j(\theta_1 + \theta_2)}$$

$$\frac{A_1 e^{j\theta_1}}{A_2 e^{j\theta_2}} = \frac{A_1}{A_2} e^{j(\theta_1 - \theta_2)}$$

Imag. part $= b_1 + b_2$

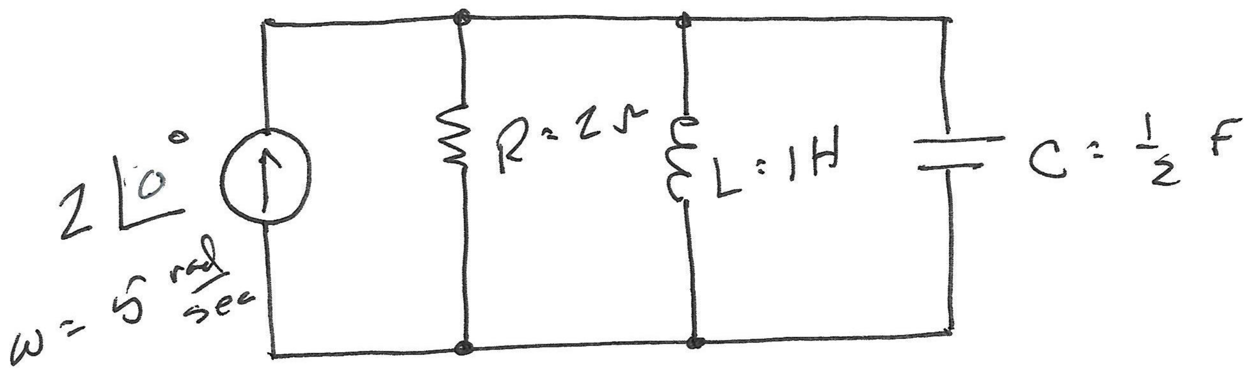


Real part of sum $= a_1 + a_2$

$$\text{Sum} = (a_1 + a_2) + j\,(b_1 + b_2)$$

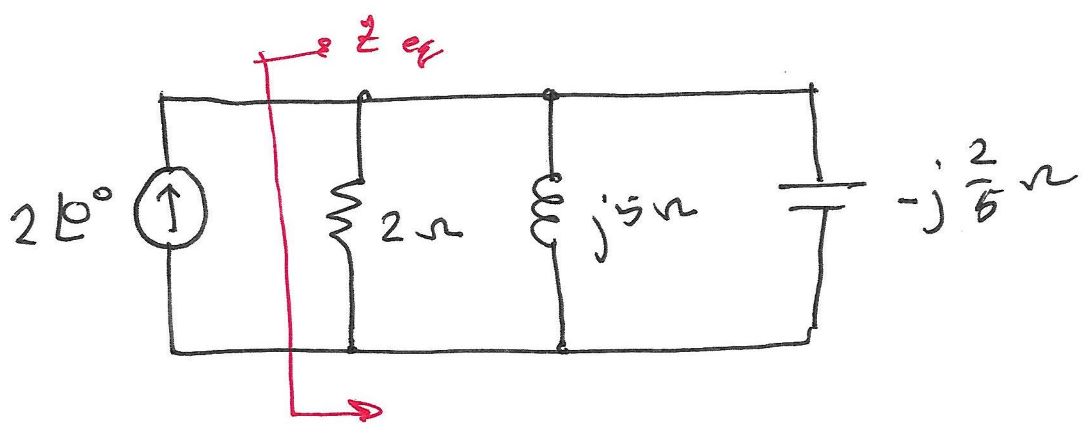
$$\text{magnitude} = \sqrt{(a_1 + a_2)^2 + (b_1 + b_2^{\,2}}$$

$$\text{angle} = \tan^{-1} \frac{b_1 + b_2}{a_1 + a_2}$$



$$Z_L = j\omega L = j5 \ \Omega$$

$$Z_C = \frac{1}{j\omega C} = -j\frac{2}{5} \ \Omega$$



$$Y_{eq} = \frac{1}{2} + \frac{1}{j5} + \frac{1}{\left(-j\frac{2}{5}\right)}$$

$$= \frac{1}{2} - j\frac{1}{5} + j\frac{5}{2} \quad S$$

$$= \frac{1}{2} + j\frac{23}{10} \quad S$$

$$Z_{eq} = \frac{1}{Y_{eq}} = \frac{1}{\frac{1}{2} + j\frac{23}{10}}$$

$$= \frac{10}{5 + j23}$$

$$= \frac{10}{(25 + 23^2)^{1/2} \angle \tan^{-1}\frac{23}{5}}$$